Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 16 January 2025

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

H:1/1/1/1/

P 7 6 1 9 5 A 0 1 3 2

Pearson

1. $f(x) = 2\sec x + 6x - 3 \qquad 0 < x < \frac{\pi}{2}$

The equation $f(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

a) * For f(x)=0 to have a single root between $0.1 < \alpha < 0.2$, there must be a change of sign

1 Substitute the values 0.1 and 0.2 in f(x)

$f(x) = 2\sec x + 6x - 3$

$2\sec(0.1) + 6(0.1) - 3$

$2 \times \frac{1}{\cos(0.1)} + 6(0.1) - 3$

$= -0.39$

$f(x) = 2\sec(0.2) + 6(0.2) - 3$

$= 2 \times \frac{1}{\cos(0.2)} + 6(0.2) - 3$

$= 0.24$

You can rewrite $\sec x = \frac{1}{\cos x}$

2 Always remember to write down the final statement!

There is a change of sign between 0.24 and -0.39, continuous and hence there is a root

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Question 1 continued

b) ① Set $f(x) = 0$

$2\sec x + 6x - 3 = 0$

You can rewrite $\sec x = \frac{1}{\cos x}$

$2 \times \frac{1}{\cos x} + 6x - 3 = 0$

$\frac{2}{\cos x} + 6x - 3 = 0$

$6x = 3 - \frac{2}{\cos x}$

$\therefore x = \frac{1}{2} - \frac{1}{3\cos x}$ thus proven

c)i) You can set ans (in your calculator) as 0.15 and substitute it into $x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$

$x_1 = 0.15$

$\therefore x_{1+1} = \frac{1}{2} - \frac{1}{3\cos x_{n+1}}$

$x_2 = \frac{1}{2} - \frac{1}{3\cos(0.15)}$

$\therefore x_2 = 0.162881\ldots$

$\therefore x_2 = 0.1629$

ii) Because we know $0.1 < \alpha < 0.2$

$\alpha = 0.1622$

(Total for Question 1 is 6 marks)

2. The weed on the surface of a pond is being monitored.

The surface area of the pond covered by the weed, A m^2, is modelled by the equation

$$\log_{10} A = 1 + 0.03t$$

where t is the number of weeks after monitoring began.

Use the equation of the model to answer parts (a) and (b).

(a) Find the surface area of the pond initially covered by the weed.

(1)

After T weeks, 25 m^2 of the pond is covered by the weed.

(b) Find the value of T, giving your answer to 2 decimal places.

(2)

a) The surface area initially will be when t=0, so we can substitute in t as 0 in our model

$\log_{10} A = 1 + 0.03(0)$

$\log_{10} A = 1$

$A = 10\text{m}^2$ $\quad\therefore A = 10\text{m}^2$

b) We now have the area (25) and we need to work out T. We can do this by substituting in A in our model equation

$\log_{10} 25 = 1 + 0.03T$

$\downarrow$

$1.39794\ldots = 1 + 0.03T$

$0.39794 = 0.03T$

$\therefore T = 13.26$ $\quad\therefore T = 13.26$

P 7 6 1 9 5 A 0 4 3 2

Question 2 continued

(Total for Question 2 is 3 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x+1}{(x+3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

This is a quotient (which is the fancy name for a fraction)

Use calculus to find the range of values of x for which y is increasing. This means that $\frac{dy}{dx} > 0$

(6)

We first have to differentiate our curve

① Looking at the equation, we can identify this as a quotient and there we must use the quotient rule for differentiation

$$y = \frac{4x+1}{(x+3)^2} \quad \begin{matrix}(u)\\(v)\end{matrix}$$

Quotient Rule:

For $y = \frac{u}{v}$,

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Note: this is given in the data sheet!

$$\frac{du}{dx} = 4$$

$$\frac{dv}{dx} = 2(x+3)$$

$$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x+3)^2(4) - ((4x+1)\,2(x+3))}{((x+3)^2)^2}$$

② Now, simplify further and remember to include $\frac{dy}{dx} > 0$

$$\frac{dy}{dx} > \frac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$$

$$\frac{dy}{dx} > \frac{4(x+3)^2 - 2(4x+1)(x+3)}{(x+3)^4}$$

$$0 > 4\underbrace{(x+3)^2}_{x^2+6x+9} - 2\underbrace{(4x+1)(x+3)}_{4x^2+13x+3}$$

$$0 > 4(x^2+6x+9) - 2(4x^2+13x+3)$$

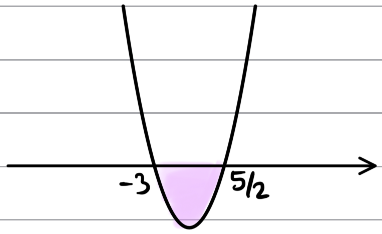
$$0 > 4x^2 + 24x + 36 - (8x^2 + 26x + 6)$$

$$0 > 4x^2 + 24x + 36 - 8x^2 - 26x - 6$$

Question 3 continued

$-4x^2 - 2x + 30 < 0$

$(2x-5)(x+3) < 0$



$\therefore -3 < x < 5/2$

(Total for Question 3 is 6 marks)

4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\, dx$$

This is definite integration as we are given the bounds

giving your answer in the form $p + q \ln 2$, where p and q are integers.

(5)

a) i) ① We can first multiply both sides by (x^2+4)

$$\frac{4x^3+2x^2+3x+8}{x^2+4} \equiv Ax+B+\frac{Cx+D}{x^2+4}$$

$\times(x^2+4)$ $\quad\quad$ $\times(x^2+4)$

$$\therefore 4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4) + (Cx+D)$$

② Now, we can expand and simplify the brackets

$(Ax+B)(x^2+4)$

$Ax^3 + 4Ax + Bx^2 + 4B$

③ And then add the $Cx + D$ (to get the expression into the correct format which matches the left side)

$Ax^3 + Bx^2 + (4A+C)x + (4B+D)$

④ We can then compare the coefficients

$Ax^3 + Bx^2 + (4A+C)x + (4B+D)$

and $4x^3 + 2x^2 + 3x + 8$

$\therefore A=4$ $\quad$ $\therefore B=2$ $\quad$ $4A+C=3$ $\quad$ Part ii) $4B+D=8$

$4(4)+C=3$ $\quad$ $4(2)+D=8$

$16+C=3$ $\quad$ $8+D=8$

$\therefore C=-13$ $\quad$ $\therefore D=0$

$\therefore A=4$

$B=2$

$C=-13$

Part ii) $D=0$

P 7 6 1 9 5 A 0 8 3 2

Question 4 continued

b) ① First, we can rewrite the integral with the values of A, B, C and D

$$\int_1^4 4x + 2 - \frac{13x + 0}{x^2+4}$$

$$= \int_1^4 4x + 2 - \frac{13x}{x^2+4}$$

General Integration:

$$\int_y^x A^b = \left[\frac{A^{b+1}}{b+1}\right]_y^x$$

② We can now integrate!

$$\int_1^4 4x + 2 - \underbrace{\frac{13x}{x^2+4}}$$

$$\frac{4x^2}{2} + 2x$$

This can be written as

$$-\frac{13}{2}\ln(x^2+4)$$

$$= 2x^2$$

because of the rule: $\int \frac{1}{x} = \ln(x)$

$\therefore$ We can write this as:

$$\left[2x^2+2x-\frac{13}{2}\ln(x^2+4)\right]_1^4$$

③ Now we can substitute our bounds in

$$= \left[2(4)^2 + 2(4) - \frac{13}{2}\ln(4^2+4)\right] - \left[2(1)^2 + 2(1) - \frac{13}{2}\ln(1^2+4)\right]$$

$$= 36 + \frac{13}{2}\ln\frac{1}{4}$$

$$= 36 - 13\ln 2$$

$\therefore = 36 - 13\ln 2$

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

(d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

a) The initial temperature is when t=0

$H = 280e^{-0.05(0)} + 24$

$\therefore H = 304°C$ $\qquad \therefore$ Initial temperature = 304°C

b) Asymptote = 24

c) We can substitute H as 144 to solve for t

$144 = 280e^{-0.05t} + 24$

$144 - 24 = 280e^{-0.05t}$

$120 = 280e^{-0.05t}$

$\frac{120}{280} = e^{-0.05t}$

$\therefore \frac{3}{7} = e^{-0.05t}$

$\ln\left(\frac{3}{7}\right) = -0.05t$

$\therefore t = \frac{\ln 3/7}{-0.05}$

$= 16.9459\ldots$ $\qquad \therefore t = 16.95$

Question 5 continued

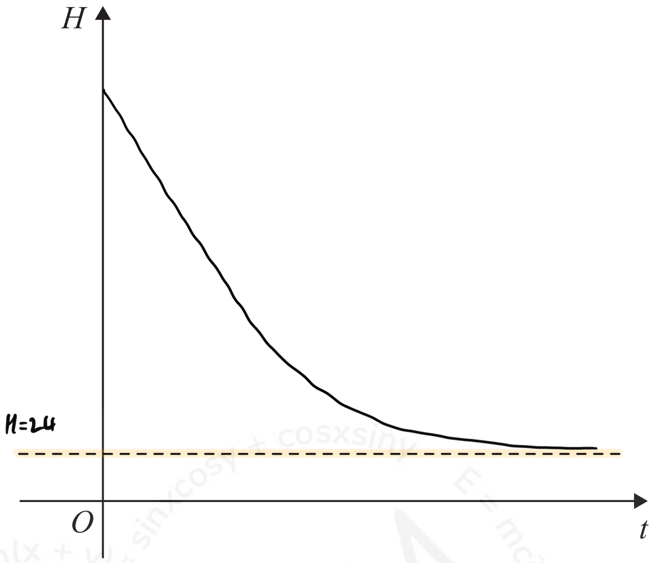


Diagram 1

Differentiating Exponentials:
$\frac{d}{dt} Ae^{-kt} = (-k)Ae^{-kt}$

d) ① We can rewrite the original equation as follows:

$H = 280e^{-0.05t} + 24$

$H - 24 = 280e^{-0.05t}$

② Now differentiate and rewrite as required

$\frac{dH}{dt}$ $280e^{-0.05t} + 24$

Remember, differentiating a constant like 24 here gives us nothing

$\frac{dH}{dt} = (-0.05) \times \underbrace{280e^{-0.05t}}_{H-24}$

$\therefore \frac{dH}{dt} = -0.05 \times (H-24)$

$= -0.05H + 1.2$

$\therefore \frac{dH}{dt} = 1.2 - 0.05H$

where $a = 1.2$ and $b = -0.05$

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The function f is defined by

$$f(x) = \frac{4x+3}{x-2} \qquad x \neq 2$$

(a) Find f^{-1}

(3)

(b) Show that

$$ff(x) = \frac{ax+b}{cx+d}$$

where a, b, c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = f(x)$.

(c) Find the point to which P is mapped when $y = f(x)$ is transformed to the curve with equation $y = 2f(3x) + 8$

(2)

a) To find the inverse, we can write y in place of x and rearrange the equation as follows:

$$\frac{4y+3}{y-2} = x$$

$$\therefore 4y+3 = x(y-2)$$

$$4y+3 = xy - 2x$$

$$4y - xy = -2x - 3$$

$$y(4-x) = -2x-3$$

$$\therefore y = \frac{-2x-3}{4-x} \qquad \therefore f^{-1} = \frac{2x+3}{x-4}$$

this can also be written as:

$$\frac{2x+3}{x-4}$$

b) ① To come ff(x), we have to substitute our original f(x) into the x values of f(x)

$$ff(x) = \frac{4\left(\frac{4x+3}{x-2}\right)+3}{\frac{4x+3}{x-2}-2}$$

② Now we can simplify!

$$= \frac{4(4x+3)+3(x-2)}{4x+3-2(x-2)}$$

$$= \frac{16x+12+3x-6}{4x+3-2x+4} = \frac{19x+6}{2x+7} \qquad \therefore ff(x) = \frac{19x+6}{2x+7}$$

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Question 6 continued

c) ① First, we have to identify the transformations happening

$y = 2f(3x) + 8$ Moving vertically by 8

Vertical stretch by scale factor 2

Horizontal stretch by scale factor $\frac{1}{3}$

② Apply the transformations to the point P (3, 15)

Horizontal stretch by scale factor $\frac{1}{3}$ → ∴ Divide x coordinate by 3

$\therefore \frac{3}{3} = 1$

Vertical stretch by scale factor 2 → ∴ Multiply y coordinate by 2

$15 \times 2 = 30$

Moving vertically by 8 → ∴ Add 8 to the new y-coordinate

$30 + 8 = 38$

∴ New coordinates of point $P = (1, 38)$

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y-axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b.

(2)

a)i) ① Find the minimum point using $y=0$

$y = |3x - a|$

$0 = 3x - a$

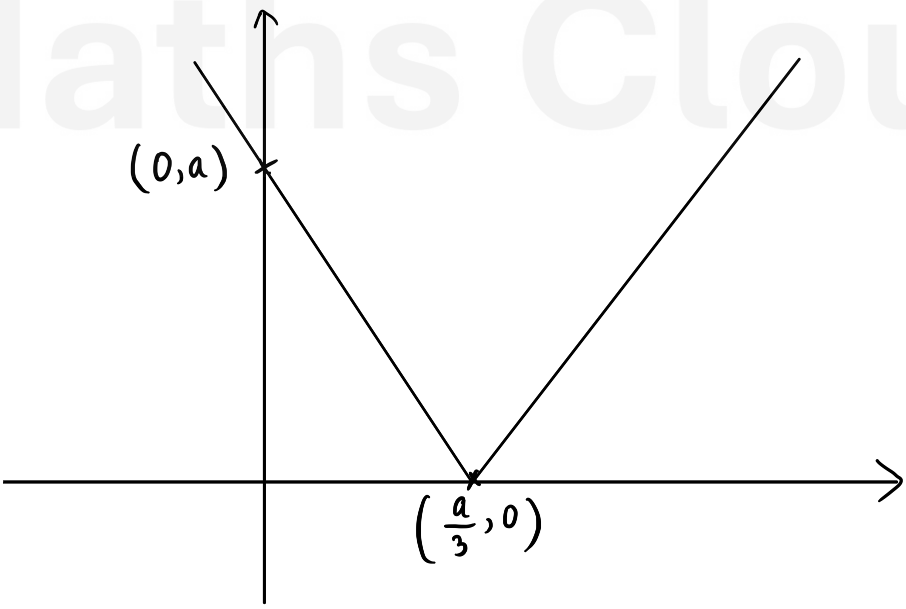
$\therefore x = \frac{a}{3}$ $\quad\therefore$ minimum point $= (\frac{a}{3}, 0)$

② Find the y intercept using $x=0$

$y = |3(0) - a|$

$y = a$ (since $a>0$)

$\therefore$ y-intercept $= (0, a)$



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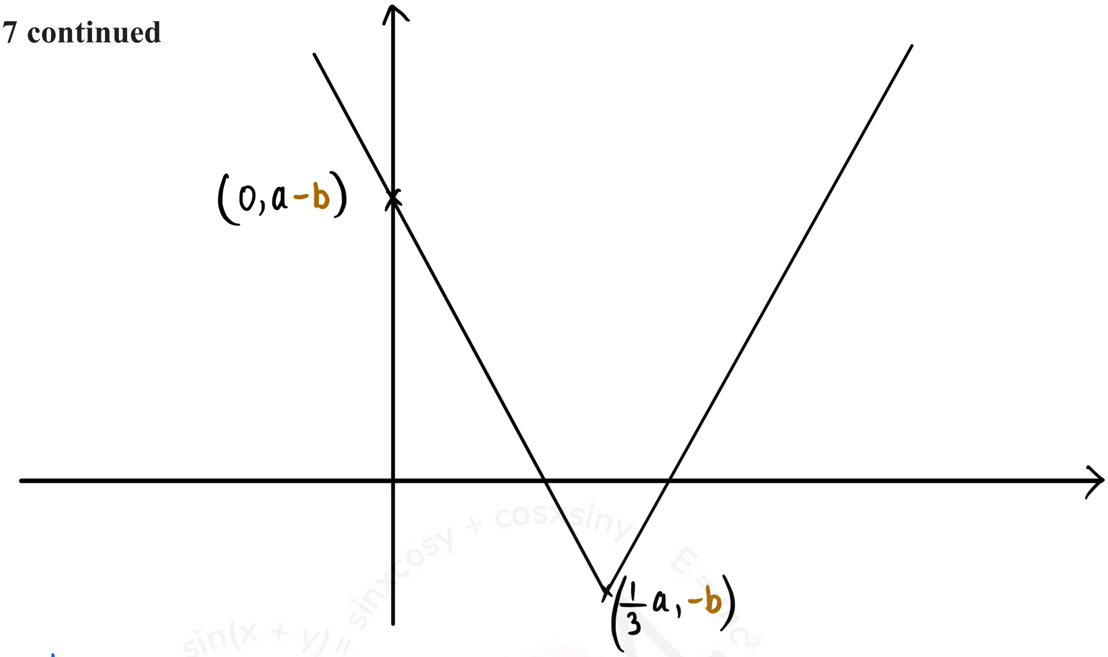
Question 7 continued



ii) $y=|3x-a|-b$ is just the previous graph shifted down by b

$\therefore$ minimum point $= \left(\frac{a}{3}, -b\right)$

Calculate the y intercept by setting x=0

$y=|3(0)-a|-b$

$y=|-a|-b$

$= a-b$

$\therefore$ y-intercept $=(0, a-b)$

b) Solve:

$\underbrace{|3x-a|}-5$

can be written as: $-3x+a$

$-3x+a-b=5x$

$a-b=8x$

$\therefore x=\frac{a-b}{8}$

$\therefore x=\frac{a-b}{8}$

(Total for Question 7 is 8 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3\,\text{cosec}\,\theta = 8\cos\theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180°$

$$\frac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

i) ① Rewrite cosec as its inverse trigonometry function

$3\,\text{cosec}\,\theta \rightarrow 3 \times \frac{1}{\sin\theta} = \frac{3}{\sin\theta}$

Reciprocal Trig Function: $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

② Simplify further:

$\frac{3}{\sin\theta} = 8\cos\theta$

Double Angle Identity: $\sin 2\theta = 2\sin\theta\cos\theta$

$3 = \underbrace{8\cos\theta\sin\theta}_{4\sin 2\theta}$

since $2\theta = 2\sin\theta\cos\theta$

$\therefore 8\cos\theta\sin\theta = 4\sin 2\theta$

$\therefore 3 = 4\sin 2\theta$

③ Now we can solve the equation (remember it's in radians!)

$3 = 4\sin 2\theta$

$\sin 2\theta = \frac{3}{4}$

$2\theta = \sin^{-1}\left(\frac{3}{4}\right) = 0.848$ AND $2\theta = \pi - 0.848 = 2.294$

$\therefore \frac{0.848}{2} = 0.424$

$\frac{2.294}{2} = 1.147$

$\therefore \theta = 0.424$

$\theta = 1.15$

P 7 6 1 9 5 A 0 2 2 3 2

Question 8 continued

ii) ① We have to identity that we must use the tan compound angle identity

$$\therefore \frac{\tan 2x - \tan 70}{1 + \tan 2x \tan 70} \Rightarrow \tan(2x - 70)$$

Compound Angle Tan Identity:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

② Now solve as normal

$$\tan(2x - 70) = -\frac{3}{8}$$

$$2x - 70 = \tan^{-1}\left(\frac{-3}{8}\right)$$

$$2x - 70 \approx -20.6$$

$$2x = -20.6 + 70$$

$$2x = 49.4$$

$x = 24.7°$ → To find the second solution, we can add $90°$

$$24.7 + 90° = 114.7°$$

$$\therefore \theta = 24.7°$$

$$\theta = 114.7°$$

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9.

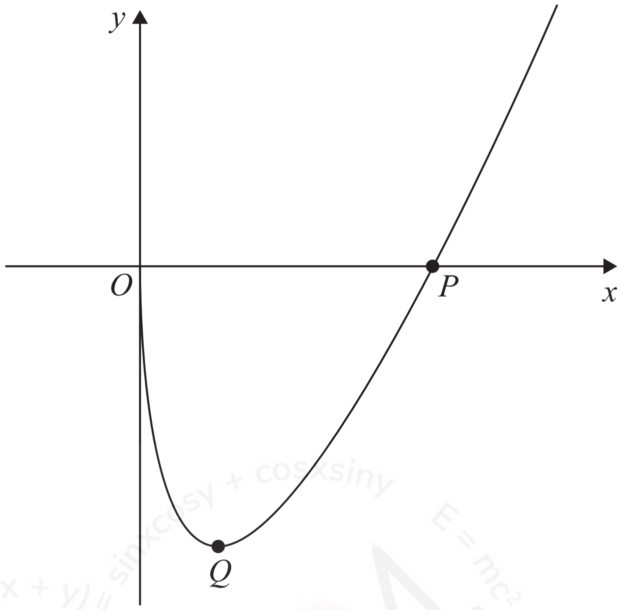


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

When $\frac{dy}{dx}=0$

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

a) To find the x-coordinate, we substitute in y=0 and then solve

$0 = 6\sqrt{x}\ln(4x)$

as $6\sqrt{x} \neq 0$

we can say $\ln(4x) = 0$

$4x = 1$

$\therefore x = \frac{1}{4}$ $\qquad \therefore x$ coordinate $= \frac{1}{4}$

P 7 6 1 9 5 A 0 2 6 3 2

Question 9 continued

b) ① The stationary point is when $\frac{dy}{dx}=0$. We have to differentiate f(x) first using the product rule

$$f(x) = \underbrace{6\sqrt{x}}_{u}\ \underbrace{\ln(4x)}_{v}$$

Product Rule

$$\frac{dy}{dx} = vu' + uv'$$

$$u' \rightarrow x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$v' \rightarrow \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = vu' + uv'$$

$$\ln(4x)\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left(\frac{1}{x}\right)$$

② Remember to put the 6 back in and then further simplify

$$6\left[\ln(4x)\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left(\frac{1}{x}\right)\right]$$

$$\therefore = 6\left[\frac{\ln(4x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right]$$

$\ast$ Note: $\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$

Factorise :

$$= \frac{3}{\sqrt{x}}\left(\ln(4x) + 2\right)$$

③ Now we can set $\frac{dy}{dx}=0$ and then solve to find the x and y coordinates

$$0 = \frac{3}{\sqrt{x}}\left(\ln(4x) + 2\right)$$

$$\therefore \ln(4x) + 2 = 0$$

$$\ln(4x) = -2$$

$$4x = e^{-2}$$

$$\therefore x = \frac{e^{-2}}{4}$$

can be written as

$$x = \frac{1}{4e^2}$$

Question 9 continued

④ Now we can find the y coordinate by substituting our x value in the original f(x) equation

we already know $\ln(4x) = -2$

$\therefore \sqrt{x} = \sqrt{\frac{e^{-2}}{4}} = \frac{e^{-1}}{2}$

$\therefore y = 6\left(\frac{e^{-1}}{2}\right) \times (-2)$

$= -6e^{-1}$

$= \frac{-6}{e}$

$\therefore Q = \left(\frac{e^{-2}}{4}, \frac{-6}{e}\right)$

c) We know that g(x) = -2f(x) and we already know the minimum of f(x) occurs at Q

$y_{min} = \frac{-6}{e}$

If $g(x) = \underbrace{-2}f(x)$

$\therefore$ y values are multiplied by -2

$\therefore -2 \times \frac{-6}{e} = \frac{12}{e}$

$\therefore$ Range $g(x) \leqslant \frac{12}{e}$

Question 9 continued

(Total for Question 9 is 8 marks)

10.

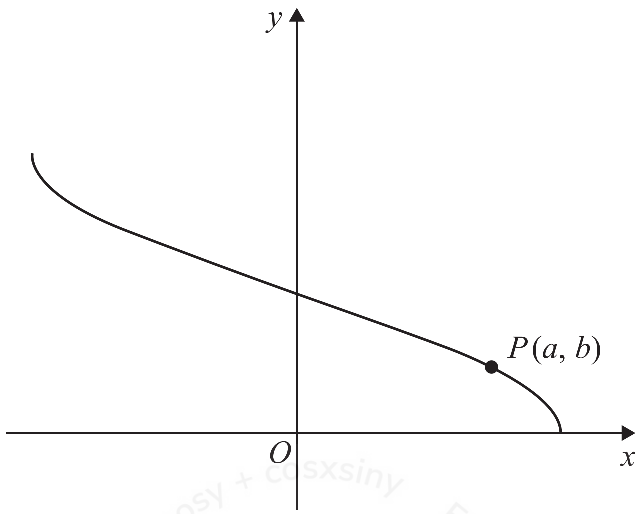


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

a) To differentiate 3cos2y, we can use our knowledge of trigonometric differentiation

$x = 3\cos 2y$

$\frac{dy}{dx} = 2(3 - \sin 2y)$

$= -6\sin 2y$ $\qquad \therefore \frac{dy}{dx} = -6\sin 2y$

Differentiating sin and cos

$\int$ sin
cos
−sin
−cos $\frac{dy}{dx}$

Question 10 continued

b) ① In part a, we found $\frac{dx}{dy}$ And now we need $\frac{dy}{dx}$. We can do this by doing $\frac{1}{\frac{dx}{dy}}$

$\therefore \frac{dy}{dx} = \frac{1}{-6\sin 2y}$

② Now we can express sin2y in terms of x

$x = 3\cos 2y$

$\cos 2y = \frac{x}{3}$

Trignometry Identity: $\sin^2 x + \cos^2 y = 1$

using $\sin^2(2y) = 1 - \cos^2(2y)$

$\sin(2y) = \sqrt{1 - \left(\frac{x}{3}\right)^2} = \sqrt{\frac{9-x^2}{9}} = \frac{\sqrt{9-x^2}}{3}$

Substitute:

$\frac{dy}{dx} = -\frac{1}{6\left(\frac{\sqrt{9-x^2}}{3}\right)}$

$= -\frac{1}{2\sqrt{9-x^2}}$

$\therefore \frac{dy}{dx} = \frac{-1}{2\sqrt{9-x^2}}$ where $k = \frac{-1}{2}$

b) ① Equate our gradient equation and $-\frac{1}{4}$ and solve for x

$\frac{dy}{dx} = \frac{-1}{4}$

$\therefore -\frac{1}{2\sqrt{9-x^2}} = \frac{-1}{4}$

cross multiply

$\therefore \frac{1}{2\sqrt{9-x^2}} = \frac{1}{4}$

$4 = 2\sqrt{9-x^2}$

$2 = \sqrt{9-x^2}$

$4 = 9 - x^2$

$x^2 = 5$

$\therefore x = \sqrt{5}$ (since its positive) $\therefore a = \sqrt{5}$

P 7 6 1 9 5 A 0 3 1 3 2

Question 10 continued

② Now solve for y using $x=3\cos 2y$

$\sqrt{5} = 3\cos 2y$

$\therefore \cos 2y = \frac{\sqrt{5}}{3}$

$2y = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$\therefore y = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$

exact values

$\therefore b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{2}\right)$

(a) (b)

$\therefore$ Point $P = \left(\sqrt{5}, \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{2}\right)\right)$

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS